2016 1st International Conference on Information Technology, Information Systems and Electrical Engineering (ICITISEE), Yogyakarta, Indonesia

A Parameter Identification Method for Fractional Order Inductance of Iron Core Reactor

Pengcheng Xia
College of Geophysics and Information Engineering
China University of Petroleum Beijing
Changping District Beijing, China
1036132089@qq.com

Zhishan Liang
College of Geophysics and Information Engineering
China University of Petroleum Beijing
Changping District Beijing, China
lzs1960@cup.edu.cn

***Abstract*—This paper is based on the capacitance and inductance are the fractional order, using the experimental method of measuring the impulse response characteristics of the iron core reactor. According to the fractional calculus Mittag-Leffler function of the Laplasse transform formula, the use of MATLAB toolbox in the MLF function program, so as to calculate the pulse response characteristics of the fractional order iron core reactor. Finally, the three parameters (equivalent resistance, inductance and fractional order) of the impedance model of an iron core reactor are obtained by the least square method. The pulse responses using the extracted parameters show very close agreement with the simulated and experimental datasets; with less than 0.1% relative error for the simulations and less than 10% from the experimental results.This method makes up for the defects of the parameters in the production design and the practical application of the iron core reactor.**

Keywords*—fractional order; impedance; measuring method; iron core reactor; impulse response

I. INTRODUCTION

Modern physics and materials science has found that the real physical system is often a very complex genetic effects and long memory effect [1]. Therefore, they often show blocking effect, post effect and non local, especially ferromagnetic material [2-3], viscoelastic material [4], electric polarization process [5-6], heat conduction process [7-8], neuron model [9], etc... Unfortunately, the classical integer order differential equation theory is not enough to describe these phenomena completely, or the model described is very complex, and it is difficult to analyze [10-11], fractional order can play a certain role here.

The iron core reactor is mainly used for reactive power compensation in power system, which plays an important role in reducing the system fault and improving the quality of the system. But when the iron core appears saturation, the magnetic field will appear nonlinear change, resulting in the inductance nonlinearity. The fractional order model can describe the nonlinear process of internal loss more than the integer order model, which can be used to describe the internal losses more than the integer order model [12]. However, in the design and production of traditional iron core reactor, the physical parameters of these stray factors are not given. This makes it difficult to control the linear interval of the iron core reactor and the inductance parameter is too small to be controlled in practical application [13].

Supported by Beijing Natural Science Foundation（3162025）.

In 2008, Ingo Schäfer and Klaus Krüger used $RL_{\beta}C$ (resistance, fractional order inductance and integer order series capacitor series) to do the resonant frequency response experiment, the extraction of the inductance component of the fractional order model parameters [12]. However, they used the standard capacitance element, the capacitance parameters as integer order; however, the authors of this paper show that: only the ideal capacitor element is integer order, the actual capacitance components are all fractional order. That is to say, it is impossible to find an ideal integer order capacitance element to measure the inductance parameter of fractional order in the experiment. Therefore, it is wrong to use the capacitance parameters as an integer order for the derivation and calculation of the formula. As a result, it is possible to ignore the nature of the actual existence of the fractional order in [12], and their analysis results may not be very accurate, the results are likely to have a large deviation.

Therefore, in this paper, based on the capacitance and inductance are the fractional order, using the experimental method of measuring the impulse response characteristics of the iron core reactor. According to the fractional calculus Mittag-Leffler function of the Laplasse transform formula, the use of MATLAB toolbox in the MLF function program, so as to calculate the pulse response characteristics of the fractional order iron core reactor. Finally, the three parameters (equivalent resistance, inductance and fractional order) of the impedance model of an iron core reactor are obtained by the least square method.

II. FRACTIONAL IMPEDANCE MODEL OF IRON CORE REACTOR

A fractional-order impedance model of iron core reactor is shown in *Fig.1*, which is composed of a series resistor R_0, and constant phase element (CPE). The CPE's impedance is given as $Z_{CPE} = (j\omega)^{\beta} L$ or $s^{\beta} L$ in the s-domain, where L is the inductance and β is its order, where when β= 0 the CPE is an ideal resistor and when β=1 an ideal inductor. While $\beta \in \Re$ is mathematically possible.

Therefore, for this paper we will limit β to the range $0 \le \beta \le 1$ .Hence, the prefactor *L* does not describe pure inductance anymore, as is also reflected by the corresponding unit $\mathrm{V}s^{\beta}\mathrm{A}^{-1}$ in [12].However, the authors found that fractional calculus is just a tool of mathematical theory, it is not possible to change the physical meaning of the inductor in the real world, inductance unit will not change, still should is Heng (H).The impedance parameter of iron core reactor is $Z = R_0 + (j\omega)^{\beta} L = Z' + jZ''$, and the time constant is $\tau = (\frac{L}{R_0})^{1/\beta}$.

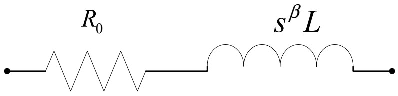

Fig. 1. Fractional impedance model of iron core reactor

III. Experimental Method and Numerical Calculation of Impulse Response

The pulse response experiment circuit is shown in *Fig.2*, which is composed of a current source, an external resistor R_1 and an iron core reactor, inside $R_2 = R_0$ .The current source is applied to the pulse signal, and the output current I_0 of the iron core reactor is measured.

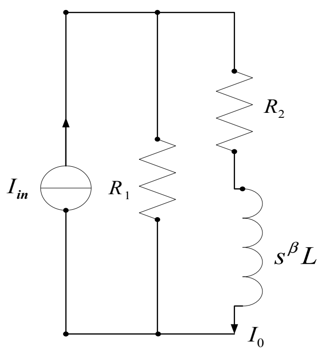

Fig. 2. Experimental circuit of impulse response

According to the experimental circuit of *Fig.2*, the calculation procedure is as follows:

Set $I_{in}(s) = I_{cc} \bullet 1$ .In *Fig.2* the circuit when time is $t \to 0^+$, by the Ohm's law

$$i_o(0^+) = \lim_{t \to 0^+} i_o(t) = \frac{R_1}{R_1 + R_2 + j\omega L} I_{cc} \quad (1)$$

$$R_2 = \frac{I_{cc}}{i_o(0^+)} R_1 - R_1 - j\omega L \quad (2)$$

In *Fig.2* circuit when time is $t \in [0, +\infty)$, by the formula of shunt shunt impedance

$$i_o(s) = R_1 I_{cc} \frac{1}{R_1 + R_2 + s^{\beta} L} \quad (3)$$

The two parameter Mittag-Leffler function is defined as follows:

$$E_{\alpha,\beta}(\mathrm{z}) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}. \quad \alpha > 0, \beta > 0, z \in C. \quad (4)$$

Laplasse transformation of two parameters Mittag-Leffler:

$$\mathcal{I}\{t^{\beta-1} E_{\alpha,\beta}(-\lambda t^{\alpha})\} = \frac{s^{\alpha-\beta}}{s^{\alpha} + \lambda}. \quad \mathrm{Re}(s) > |\lambda|^{\frac{1}{\alpha}} \quad (5)$$

Among them $t \geq 0$, s is variable in the Laplace domain, $\mathrm{Re}(s)$ defines the real part of s, $\lambda \in R$.

In *Fig.2* circuit when time is $t \in [0, +\infty)$, by the Mittag-Leffler function of the Laplasse transforms formula

$$\begin{aligned} i_o(t) &= \mathcal{I}^{-1}[\frac{R_1 I_{cc}}{R_1 + R_2 + s^{\beta} L}] \\ &= \frac{R_1 I_{cc}}{L} \mathcal{I}^{-1}[\frac{1}{s^{\beta} + \frac{R_1 + R_2}{L}}] \\ &= \frac{R_1 I_{cc}}{L} t^{\beta-1} E_{\beta,\beta}(-\frac{R_1 + R_2}{L} t^{\beta}) \end{aligned} \quad (6)$$

The definition of the function by Mittag-Leffler

$$i_o(t) = \frac{R_1 I_{cc}}{L} t^{\beta} \sum_{k=0}^{\infty} \frac{(-\frac{R_1 + R_2}{L} t^{\beta})^k}{\Gamma(\beta k + \beta)} \quad (7)$$

The gamma function is defined as follows:

$$\Gamma(\mathrm{z}) = \int_0^{\infty} e^{-t} t^{z-1} dt, \quad \mathrm{Re}(z) > 0 \quad (8)$$

The definition of substitution of gamma function

$$i_o(t) = \frac{R_1 I_{cc}}{L} t^{\beta} \sum_{k=0}^{\infty} \frac{(-\frac{R_1 + R_2}{L} t^{\beta})^k}{\int_0^{\infty} e^{-t} t^{\beta k + \beta - 1} dt} \tag{9}$$

By using MLF MATLAB toolbox (Function Mittag-Leffler) program, (6) can be calculated to obtain the core reactor output current Io pulse response curve data.

According to the experimental circuit shown in *Fig.2*, we can also use the step response method. The current source is applied to the step signal, and the calculation process is as follows:

Set $I_{in}(s) = I_{cc} / s$ .In Fig.2 the circuit when time is $t \to \infty$, by the Ohm's law:

$$i_o(\infty) = \lim_{t \to \infty} i_o(t) = \frac{R_1}{R_1 + R_2} I_{cc} \tag{10}$$

$$R_2 = (\frac{I_{cc}}{i_o(\infty)} - 1) R_1 \tag{11}$$

In *Fig.2* circuit when time is $t \in [0, +\infty)$, by the formula of shunt shunt impedance

$$i_o(s) = R_1 I_{cc} \frac{s^{-1}}{R_1 + R_2 + s^{\beta} L} \tag{12}$$

In *Fig.2* circuit when time is $t \in [0, +\infty)$, by the Mittag-Leffler function of the Laplasse transforms formula

$$\begin{aligned} i_o(t) &= \mathcal{I}^{-1}[R_1 I_{cc} \frac{s^{-1}}{R_1 + R_2 + s^{\beta} L}] \\ &= \frac{R_1 I_{cc}}{L} \mathcal{I}^{-1}[\frac{s^{-1}}{s^{\beta} + \frac{R_1 + R_2}{L}}] \\ &= \frac{R_1 I_{cc}}{L} t^{\beta} E_{\beta, \beta+1}(-\frac{R_1 + R_2}{L} t^{\beta}) \end{aligned} \tag{13}$$

The definition of the function by Mittag-Leffler

$$i_o(t) = I_{cc} \frac{R_1}{L} t^{\beta} \sum_{k=0}^{\infty} \frac{(-\frac{R_1 + R_2}{L} t^{\beta})^k}{\Gamma(\beta k + \beta + 1)} \tag{14}$$

The definition of substitution of gamma function

$$i_o(t) = I_{cc} \frac{R_1}{L} t^{\beta} \sum_{k=0}^{\infty} \frac{(-\frac{R_1 + R_2}{L} t^{\beta})^k}{\int_0^{\infty} e^{-t} t^{\beta k + \beta} dt} \tag{15}$$

According to the Thevenin's theorem and Norton's theorem, the voltage source current source equivalent substitution, the calculation process is similar.

IV. Parameter Identification Method

In *Fig.2* shows impulse response experiment method parameter measurement and numerical calculation using Matlab toolbox function of MLF program. By (6) can be calculated by iron core reactor output current *I*o impulse response curve data. The following identification algorithm is based on the least squares method, to illustrate how to measure the identification of the core reactor of the fractional order model parameters (R_0, L and β).

The study found that: from iron core reactor output current impulse response curve, pulse response experiment output current curve presented starting from zero to suddenly consecutive rises to the maximum peak, and then slowly continued to fall to zero. When the inductance is L constant, the peak current decreased with the decreasing of the order number; the rise time of peak current decreased; the slope of the curve increased. When the order of beta coefficient is constant, with the decrease of the inductance L of arithmetic, the peak current are the same, but reached peak current rise time is decreasing arithmetic.

For example, set R_2=10Ω, R_1=100Ω, I_{cc}=5A, L=1H, select β= 0.9, 0.7, 0.5, 0.3, impulse response of the output current curve as shown in *Fig.3*, starting from the initial time t = 0 s, current curve rose rapidly to reach the peak, peak current are $I_{\max}$ =1.404, 0.9673, 0.6114 ,0.1274 (A). When the inductance is constant, the peak current decreases with the decreasing of the order, the rise time decreases and the slope of the curve increases.

For example, hold R_2=10Ω, R_1=100Ω, I_{cc}=5A constant, and set β =0.863, selection of L = 1, 0.8, 0.6, 0.4(H), impulse response of output current curve as shown in *Fig.4*, starting from the initial time t = 0 s, current curve rose rapidly to reach the maximum value =1.314A, peak current are the same, but the rise time respectively is $t_{I_{\max}}$ = 1.3, 2.1 ,3.0 3.9 ms, the rise time of the displacement constant is 0.8 ms. When the order number is constant, with the decrease of the inductance of the arithmetic, the peak current remained unchanged, but the rise time is decreasing arithmetic.

The experimental results show that the magnitude of the peak current and the rise slope of the curve is determined by the number of orders; when the order is the same, the rise time is determined by the inductance. Therefore, the idea of the inductor parameter identification of the iron core reactor is as follows:

Firstly, according to the boundary conditions of the initial time of the experimental circuit of *Fig.2*, the equivalent resistance of the iron core reactor R_o.

Then, the number of identification based on the experimental circuit shown in *Fig.2*, using several parameters known standard fractional iron core reactor. The order number is several typical values, such as: 0.1, 0.2, 0.8, 0.9, and so on, measuring the standard fractional iron core reactor pulse response output current curve data, different typical order number corresponding to the peak current typ. Change into the tested iron core reactor pulse response experiment, the tested iron core reactor output current peak and typical values for comparison, measured the value range of the order of the iron core reactor, thus identification algorithm set the initial order. It's used least square optimization to get the exact value of the measured iron core reactor.

Finally, the inductance is identified. Using several order the same but different inductance standard fractional iron core reactor, the inductance is several typical values, such as: 0.1, 0.2, 0.8, 0.9, and so on, measuring the standard fractional iron core reactor pulse response output current curve data, get different typical inductance corresponds to the rise time of the typical value, by the relationship between the rise time and the inductance quantity to determine the interval of the iron core reactor inductance values to be tested, thus identification algorithm set the initial value of inductance. The inductance value of the iron core reactor is obtained by least square optimization.

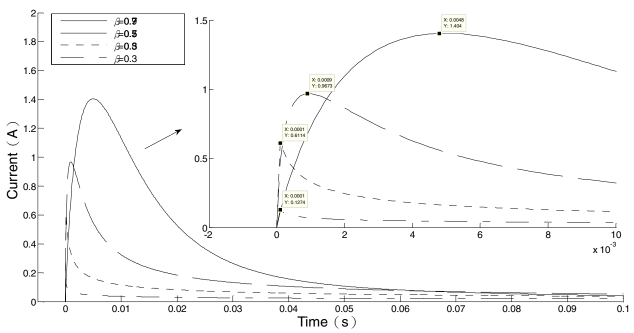

Fig. 3. Output current impulse response curve of standard iron core reactor with β variation

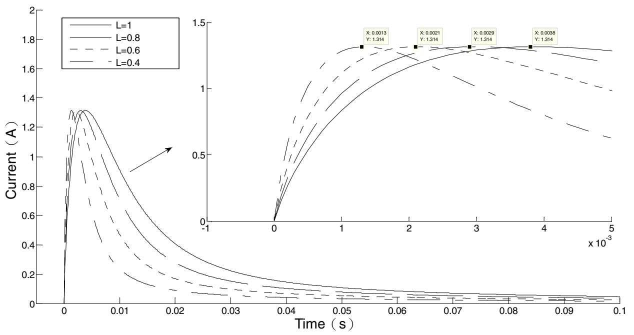

Fig. 4. Output current impulse response curve of standard iron core reactor with L variation

The parameter identification algorithm flow is as follows, as shown in *Fig.5*:

Step 1, experimental acquisition of the core reactor output current pulse response data, accessed to I_o^*.

Step 2, According to the boundary conditions of the initial time of the experimental circuit in *Fig.2*, the R_2 is determined by (2).

Step 3, the rise time of the current Io is defined as the slope of the rising current, and the calculation formula is as follows:

$$k_{i_o} = \frac{I_{90\%} - I_{10\%}}{\log_{10} t_{I_{90\%}} - \log_{10} t_{I_{10\%}}} \tag{16}$$

Step 4, given (or update) the initial value of L and β and substituted into (6), and call the MATLAB tools in MLF subroutine for $I_o(t)$ time domain analytical solution.

Step 5, judge: the pulse response of the output current of the iron core reactor is equal to the slope of the simulation curve and the experimental curve. If it is not equal, the value of the β is modified in accordance with the following formula,

$$\begin{aligned} \Delta\beta &= \beta \times 5\% \\ \beta &= \beta + \Delta\beta \end{aligned} \tag{17}$$

And then return to step 4, update the initial value of the beta, and continue to optimize;

If equal, the value of the fractional order model of the core reactor is determined, and then the next step is to continue.

Step 6, the displacement constant of the current curve calculated by the experimental data, is defined as follows:

$$\Delta T_f = \frac{t_{I_{f2}} - t_{I_{f1}}}{L_2 - L_1} \tag{18}$$

Among them, $t_{I_{f1}}$ 、 $t_{I_{f2}}$ are the corresponding time of the maximum peak current I_{f_1} 、 I_{f_2}, the calculation formula of the maximum peak current is as follows:

$$I_f = \frac{R_1}{R_1 + R_2} I_{cc} \tag{19}$$

Step 7, by the known L_1 and its experimental data, and according to (18) to obtain the L of the core reactor to be measured, the expression is as follows:

$$L = L_1 + \frac{t_{I_f} - t_{I_{f1}}}{\Delta T_f} \tag{20}$$

Step 8, judge: whether to meet the least squares optimization index (the expression is as follows),

$$\min_t \left\| i_o(t) - i_o^* \right\|_2^2 = \min_t \sum_k^n (i_o(t)_k - i_{ok}^*)^2 \qquad (21)$$

If this condition is not satisfied, the value of L is modified according to the following formula,

$$\begin{aligned} \Delta L &= L \times 5\% \\ L &= L + \Delta L \end{aligned} \qquad (22)$$

Then go back to step 4, update the initial value of the *L*, and continue to optimize;

If it is satisfied, then determine the *L* value of the core reactor to be measured.The program ends.

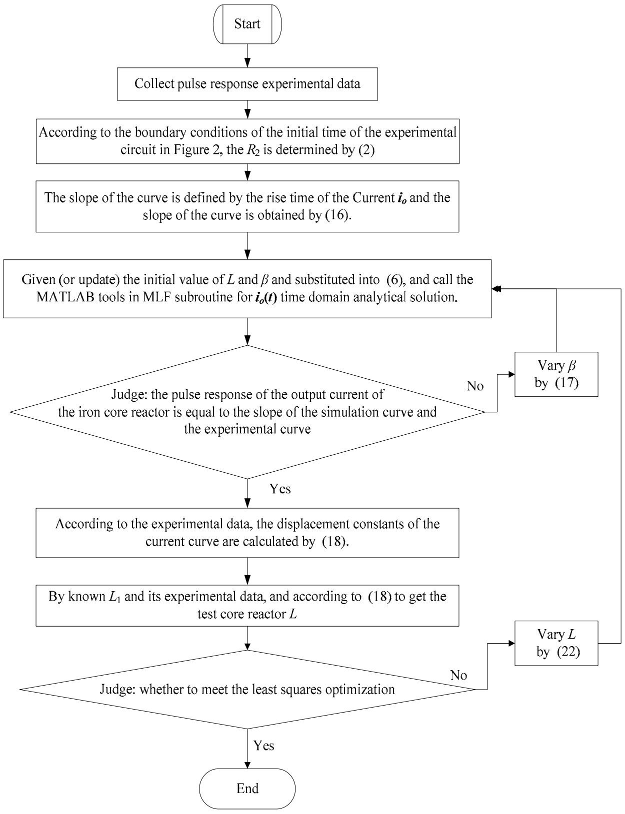

Fig. 5. Flow chart of parameter identification algorithm

Using MATLAB programming to achieve the above algorithm, the final identification results of the measured value of the inductor can be obtained,which is R_2=10Ω ,L=0.531H, the order number is β=0.863.

To verify this, the impulse response of the iron core reactor is calculated using the fractional model.In order to do so, the parameters L and β (determined above) will be used.

The results of simulation calculation of the iron core reactor output current pulse response curve data, compared with the experimental curve, are shown in *Fig.6*. From Fig.6, you can see that the agreement is excellent.

In contrast to the conventional model, the fractional model well represents the current curve through the iron core reactor.The absolute error of experiment and simulation is less than 0.1%, the relative error is less than 10%.

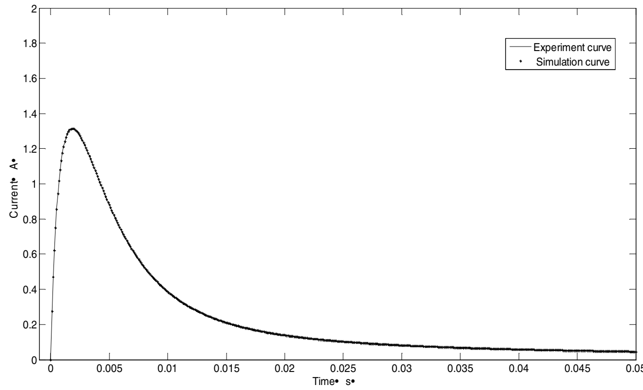

Fig. 6. The curve of the output current impulse response of the iron core reactor

V. CONCLUSION

This paper is based on that capacitance and inductance is actual understanding of the fractional order, using two standard fractional iron core reactor to do the experiment of impulse response, which parameters are known and fractional order are same. Using the measured standard fractional iron core reactor output current curve data to obtain standard fractional iron core reactor output current displacement curve constant. By measuring the impulse response data, the fractional order parameter of the core reactor is identified. In addition, the experimental identification of the step response is similar to that of the impulse response experiment. The pulse responses using the extracted parameters show very close agreement with the simulated and experimental datasets; with less than 0.1% relative error for the simulations and less than 10% from the experimental results. This method makes up for the defects of the parameters in the production design and the practical application of the iron core reactor.

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